**Supplementary File 1.** Pair-wise comparisons of evolutionary rate. For each trait (KT, input link, output link, and coupler link), the Brownian Motion evolutionary rate parameter (σ2) is given. For each comparison, the AICC score for a model in which rates are allowed to vary (obs.) and constrained to be equal (const.) are given, as are the Likelihood Ratio Test (LRT) score and corresponding *p* value for df=1.

**Cichlids**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Evolutionary rate, σ2 | KT | | Input | Output | Coupler |
|  | | 5.3e-4 | 7.9e-4 | 2.7e-4 | 4.0e-4 |
| Comparison | | AICobserved | AICconstrained | LRT | *p* |
| Input - Output | | -162.627 | -155.102 | 9.525 | 0.002 |
| Input - Coupler | | -156.213 | -153.542 | 4.671 | 0.031 |
| Output - Coupler | | -178.536 | -179.343 | 1.194 | 0.275 |

**Mantis Shrimp**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Evolutionary rate, σ2 | KT | | Input | Output | Coupler |
|  | | 1.2e-4 | 3.8e-5 | 1.7e-4 | 2.2e-5 |
| Comparison | | AICobserved | AICconstrained | LRT | *p* |
| Input - Output | | -206.227 | -189.302 | 18.925 | 1.4e-5 |
| Input - Coupler | | -383.040 | -383.936 | 2.897 | 0.089 |
| Output - Coupler | | -227.024 | -195.197 | 33.827 | 6.0e-9 |

**Sunfish**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Evolutionary rate, σ2 | KT | | Input | Output | Coupler |
|  | | 6.9e-4 | 3.7e-5 | 4.8e-4 | 1.0e-4 |
| Comparison | | AICobserved | AICconstrained | LRT | *p* |
| Input - Output | | -125.984 | -93.906 | 34.078 | 5.3e-9 |
| Input - Coupler | | -145.052 | -142.272 | 4.779 | 0.029 |
| Output - Coupler | | -95.769 | -87.164 | 10.604 | 0.001 |

**Wrasses**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Evolutionary rate, σ2 | KT | | Input | Output | Coupler |
|  | | 5.5e-3 | 1.9e-3 | 3.1e-3 | 1.5e-3 |
| Comparison | | AICobserved | AICconstrained | LRT | *p* |
| Input - Output | | -413.880 | -417.389 | 5.509 | 0.019 |
| Input - Coupler | | -346.797 | -346.405 | 1.608 | 0.205 |
| Output - Coupler | | -553.817 | -537.762 | 18.056 | 2.1e-5 |