FRAP reactions

$$C_{in}^{f} \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C_{mem}^{f} \xrightarrow{k_b} C_{mem}^{b}$$

Kinetic equations

$$\frac{\partial C_{in}^{f}}{\partial t} = \beta k_1 C_{mem}^{f} - k_1 C_{in}^{f} \text{ hence } C_{in}^{f} = \beta C_{mem}^{f} - C_0 e^{-k_1 t};$$

$$\frac{\partial C_{mem}^{f}}{\partial t} = k_1 C_{in}^{f} - \beta k_1 C_{mem}^{f} - k_b C_{mem}^{f} \text{ hence}$$

$$\frac{\partial C_{mem}^{f}}{\partial t} = -k_1 C_0 e^{-k_1 t} - k_b C_{mem}^{f} \text{ hence}$$

$$C_{mem}^{f} = C_1 e^{-k_b t} - \frac{k_1}{k_b - k_1} C_0 e^{-k_1 t};$$

Initial and boundary (limiting) conditions

$$C_{mem}^{f}\Big|_{t=0} = 0 \text{ hence } C_{mem}^{f} = \frac{k_1}{k_b - k_1} C_0 \left(e^{-k_b t} - e^{-k_1 t}\right);$$

$$C_{mem}^{f}\Big|_{k_b=0, t\to\infty} = RC_{in} \text{ hence}$$

Fluorescence kinetics

$$C_{mem}^{f} = RC_{in}\left(e^{-k_b t} - e^{-k_1 t}\right)$$